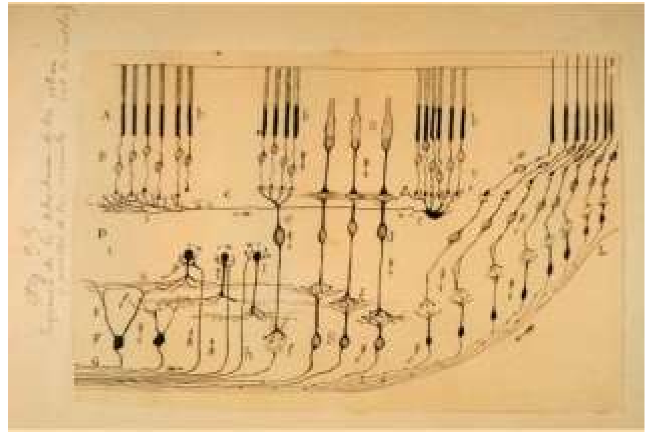
Light and Life: Geometrical Optics and Image Formation

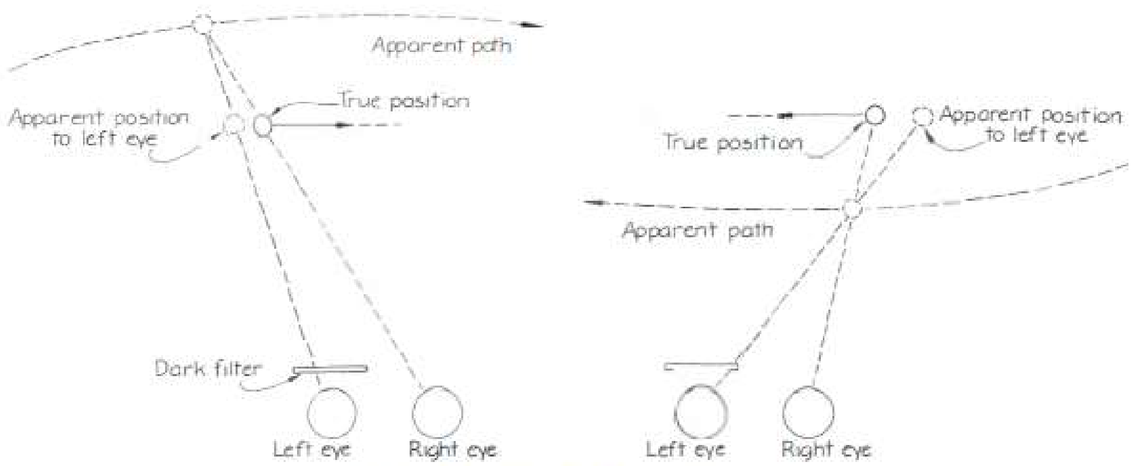
As Plato advised us in the "***Allegory of the Cave***," we can discover and learn the **rules** that relate our perception of reality to reality itself. Today we will talk about the mathematical or geometrical rules known as the **laws of reflection and refraction** that relate the perceived image to the real object. But before we do, I want to remind you of two illusions that demonstrate that the rules that relate perception to reality are not limited to those that are mathematical or geometrical. As Bishop Berkeley predicted, there are also higher-order and "wholistic" rules than cannot be **reduced** currently if ever to geometry.

Last class we experienced the **Pulfrich pendulum effect**, which taught us that our **perception** of an object is ***not*** determined solely by the physical distribution of **light energy** at a given point in **space** at a given instant in **time**. Is this illusion an evolutionary maladaptation and/or a deficiency in the design of the visual system or a consequence of evolutionary adaption and/or good design? The eye that we cover with a neutral density filter while observing the Pulfrich pendulum illusion becomes dark-adapted and the rods become the primary photoreceptors (**skotopic vision**). Rods are sensitive to minute levels of light, but the high sensitivity of the rods can result in their occasional "misfiring" in the dark. In order to prevent the perception of would-be annoying twinkling noise from this misfiring, several rods are connected to a single bipolar cell in the retina. Together, the neural cells in the retina **integrate spatially and temporally** the output from several rods so that the ganglion cell only transmits a signal to the brain when the inputs from the rods surpass a threshold. This integration increases the signal-to-noise ratio by pooling weak signals in the dark-adapted eye before sending an impulse to the brain and withholding from the brain any isolated signal that would most likely represent would-be annoying twinkling noise.



"Schematic representation of mammalian retina structure. Artistic grouping of cells and direction of current flow" A, layer of rods and cones; B, visual cell body layer; C, outer plexiform layer; D, bipolar cell layer; E, inner plexiform layer; F, layer of ganglion cells; G, optic nerve fiber layer; L, central fossa. Modified from a photograph taken from the original (22X32 cm). Drawn on sheet/paper. P.Y. 1901. S.R. y Cajal Institute - CSIC - Madrid, Spain.

As a result of the **neural integration** the dark-adapted eye sends a delayed impulse to the brain that reports on the past position of the apple while the light-adapted eye (**photopic vision**), which relies on the **cones** of the **fovea**, sends an immediate impulse to the brain that reports the nearly-present position of the apple. At any instant, the brain receives two messages—one from the light-adapted eye that gives the nearly-present position of the apple and one from the dark-adapted eye that gives the past position of the apple. The mind interprets the two messages to mean that the apple is farther away when it moves from the covered eye towards the uncovered eye and closer when the apple moves from the uncovered eye towards the covered eye.



The analogous Pulfrich illusion with a swinging pendulum

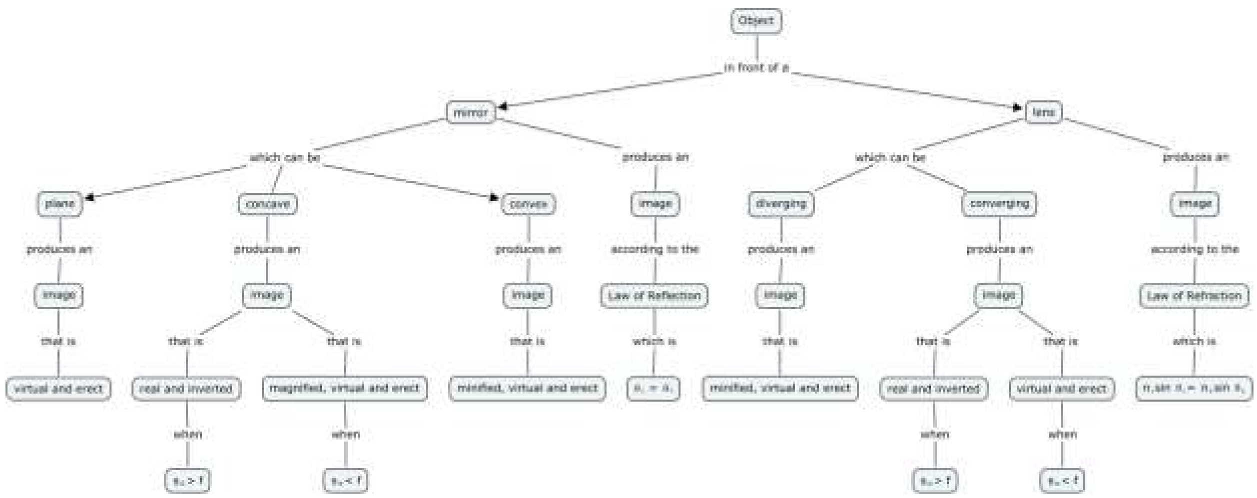
The Pulfrich pendulum effect is not necessarily a maladaptation or a design flaw, but a happy consequence of the adaptive and well-designed ability of the neural cells to "**rule**" over the rods they are connected to in order to minimize distracting twinkling. This rule increases the signal-to-noise level in dim light by performing an integration, which is similar to **freshman-level calculus**. Perhaps it is possible, as Richard Dawkins (1987) wrote in "*The Blind Watchmaker*," "*to understand [a complex process such as vision] in terms of simpler parts that we do already understand."* Taking his hierarchical reductionism to the limit, Dawkins **believes** that complex objects and processes will be "*ultimately explained in terms of the smallest of fundamental particles*."

When we see the **waterfall illusion**, it is because the neurons in the brain (at least in a cat's brain in which the electrophysiological experiments have been done) "**rule**" over what we (or the cat) perceive. When we look at a constantly moving object, such as a waterfall, the mind seems to consider it "status quo and safe" and the neurons in the brain that are involved in sensing motion adapt. Consequently, when we change the input to these neurons by looking away or stopping the motion, we temporarily perceive stationary objects as objects moving in the opposite direction. The illusion seems to be a consequence of our mind reducing the priority of constant velocity and increasing the priority of acceleration (= change in velocity). This seems reasonable to anyone who may be considered to be someone else's dinner.

It is generally true that a given cause or action has more than one effect. The Pulfrich pendulum effect leads us to the insight that higher-level rules ensure that the sensitivity of dark-adapted vision is good **enough** but not maximal, since maximal sensitivity would cause us to see annoying twinkling. The waterfall illusion leads us to the insight that everything cannot be the highest priority and higher-level rules ensure that **the mind sets priorities** in terms of perceiving movement. The **tradeoff** that occurs when a **part** of the mind cedes its ability to sense constant movement is that the **whole** mind can be more aware of potentially dangerous movement. By studying nature, we see the universal truth that **too much of a good thing may not be a good thing** and that **everything has values and limitations**.

This truth is concisely demonstrated by the **camera obscura.** We saw that when a piece of translucent vellum is put up to a window it does not produce an image even though it captures most of the image-forming light rays that originate from each point on the objects outside the window. However, when the number of image-forming rays originating from each given point on the object is limited by a pinhole, *mirable dictu*, an image is formed. In fact, the more the aperture reduces the number of image-forming rays, the sharper the image is. Thus, we see, literally and figuratively, that more of a good thing is not always better. There are tradeoffs here too, since the smaller the aperture, the dimmer the image is, and we see that less of a good thing is not always better either.

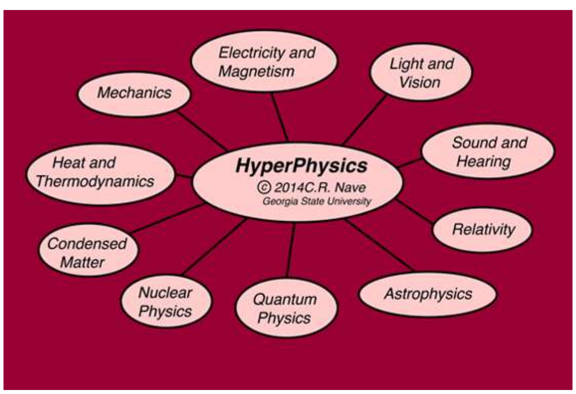
While we are talking about the mind, I want to remind you of, or introduce you to, **concept maps**, which were developed by Joe Novak at Cornell University to help students learn science in a meaningful way. A concept map is a concise representation of ideas that answers a specific **focus question**, such as: What is the relation between the object and the image according to geometrical optics? The very act of constructing a concept map develops logical thinking and a deeper understanding of the concepts because a well-constructed concept map reveals hierarchical connections between concepts and tests your understanding of how the parts relate to the whole. **Cmap Tools** is software that can facilitate your use of concept maps and is available to you at no cost at http://cmap.ihmc.us/. Here is a concept map that I just drew for the focus question: What is the relation between the object and the image according to geometrical optics?

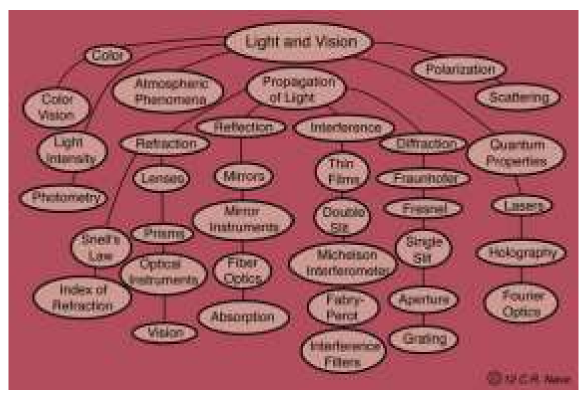


Concept maps are created by first choosing a **provisional focus question**, which is to be answered by the concept map. It is often the case that the focus question gets revised as you construct the concept map as a result of your understanding becoming more refined. The second step in forming a concept map is to think of the **concepts** that are related to the focus question. Then you do triage on the list to glean the most important concepts. In the above case, important concepts included object, image, law of reflection, mirror, law of refraction, lens, real, virtual, inverted, erect, magnified, minified, the object position (s_o) and the focal length (f). The third step is to choose **explicit linking words** that make **meaningful connections**, known as **propositions**, between the concepts. As you develop the propositions in the map, you create a **hierarchy**, through trial and error, that most clearly and completely answer your focus question.

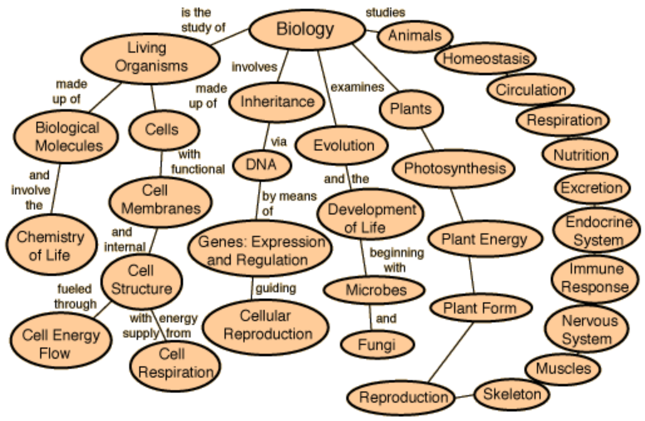
Notice that this concept map does not even mention the eye or the mind. You can create a different concept map that relates the object to the image on the retina by including concepts such as cornea, aqueous humor, pupil, iris, crystalline lens, vitreous humor, and retina. You can also create a concept map that relates the object to the image perceived by the mind's eye. Cmap Tools will allow you to link related concept maps. It will also allow you to build concept maps with other students in the class. Cmap Tools allows you to attach pictures to the concept, which is a great way of using the photographs that you take for your calendar to help you study.

The HyperPhysics website, (http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html) makes use of concept maps to explain many concepts in physics, including concepts related to light and vision (http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html).





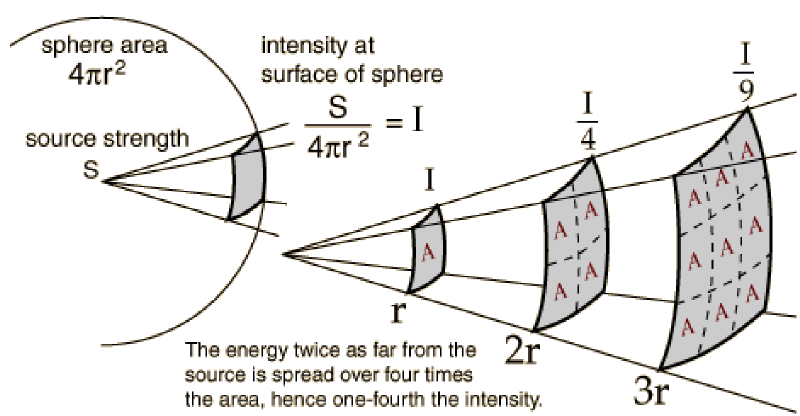
The HyperPhysics website also includes biology concepts (http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html):



The size and form of the **shadows** cast by opaque objects such as camels lit by sunlight in the desert, trees lit by moonlight, and the Pilobolus dancers lit by stage light can be most clearly and **parsimoniously described and explained** if **light travels in straight lines**.

Likewise, the **images** formed in the **camera obscura** can be most clearly and **parsimoniously described** and **explained** if **light travels in straight lines**.

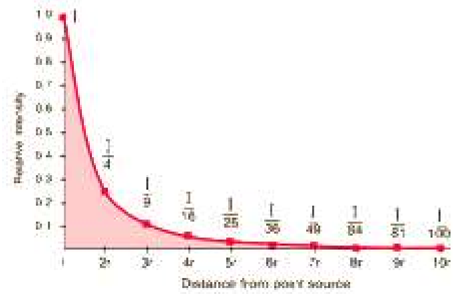
If light rays radiated in straight lines from a point source of light in all directions, the intensity of the light would decrease by the inverse square of the distance. This is because the radiant power (= energy per unit time) from the source would be spread out over a surface whose **area** is proportional to the square of the **distance** from the source. Since the surface area of a spherical surface is given by ($4\pi r^2$) and the distance is given by the radius (r), the intensity (I_r) of the light striking a constant area size



at distance (r) from the source is proportional to the radiant power of the source (S_{source}) and inversely proportional to the square of the radial distance. Let's test the **inverse square law**.

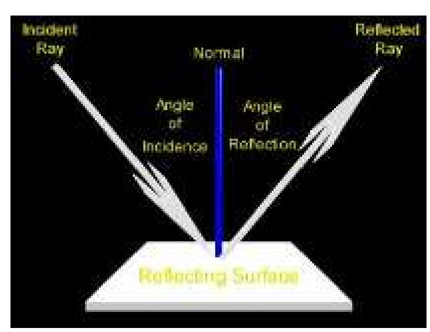
$$I_r = \frac{S_{source}}{4\pi r^2}$$

Demonstration: Use a beeswax candle from Monticello as an approximation of a point source of light and measure the intensity at successive distances with a quantum radiometer. Sketch your results and observe the shape of the curve. Are your results described by the **inverse square law**? The quantum radiometer is calibrated in µmol photons m^{-2} s^{-1} (= 6.02 x 10^{17} photons m^{-2} s^{-1}). We will assume that **corpuscles** of light, which Einstein called *lichtquanten*, and are now universally known as **photons**, are moving out from the source along each ray.



Now we will talk about the influence of **reflection** on **image formation**. We now have sufficient empirical evidence to believe that **light travels in straight lines**. However, as was known by the ancient Greek philosophers, when a light ray strikes a mirror it **changes direction** and travels as a different straight line as described by the **law of reflection**. While the mathematical law of reflection had been known for centuries, Dante Alighieri (1265-1321) poetically described the law of reflection in Pugatorio Canto XV of *The Divine Comedy*:

As when from off the water, or a mirror,
The sunbeam leaps unto the opposite side,
Ascending upwards in the self-same measure
That it descends, and deviates as far
From falling of a stone in line direct,
(As demonstrate experiment and art) . . .



…or more succintly, $\theta_r = \theta_i$, **the angle of reflection (θ_r) is equal to the angle of incidence (θ_i).**

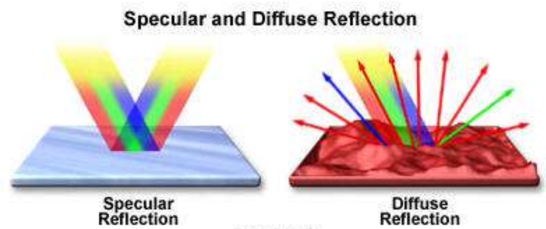
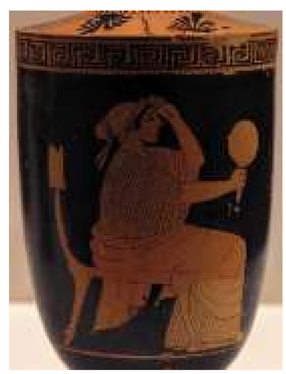
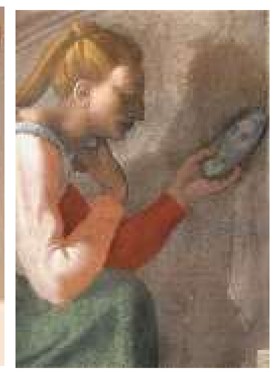


Figure 3

Sunlight is reflected from a shiny metal mirror in such a way that a beam of sunlight, composed of parallel rays is reflected at the same angle relative to the normal as the incident beam strikes the mirror. The reflected light leaves the surface as a beam and this is known as **specular reflection**. However, if the surface that the sunlight strikes is not smooth and shiny, but coarse and rough, the rays that compose the beam strike the surface at many angles and the rays of light are reflected at those many angles, each of which obeys the law of reflection. The reflected light forms a cone and this is known as **diffuse reflection**. In ancient times, the reflection from polished obsidian (6000 BC), stone or metal mirrors



was probably more diffuse than and not as specular as the reflection from mirrors we use today made of flat metal-coated glass. http://www.ucl.ac.uk/museums-static/digitalegypt//metal/mirrors.html

Primitive blown glass mirrors coated with metal were developed in Sidon in 1 AD. Since blown glass is not as flat as plate glass, these mirrors were small and not very plane. Metal-coated glass mirrors improved. In his encyclopedic book, written in the 13th century entitled, ***Speculum Majus*** (which is Latin for Large Mirror) that "*contains that [which] is worth admiring or imitating among the things that have been done or said, in the world visible and invisible*," **Vincent of Beauvais** (1190-1264) extolled the virtues of tin-coated glass mirrors over polished metal mirrors. By the 16th century, reflective and flat mercury-coated **plate glass** mirrors were produced by Venetian glassmakers. The secret process used to make the excellent mirrors became known throughout Europe as a result of industrial espionage. The toxic mercury was finally replaced with silver, tin and/or aluminum.

At the time the New Testament was written, the images produced by mirrors (*specula* in Latin) were probably not very sharp. Moreover, the mirrors were probably made of metal and not glass. Consequently, "*For now we see only a reflection as in a mirror*" was probably anachronistically translated in the 17th century as "*For now we see through a glass, darkly* (*Videmus nunc per speculum in aenigmate)*" (1 Corinthians 13:12).

As **Roger Bacon** (1214-1292) realized, the Bible makes use of many optical analogies involving reflections. It is written in Proverbs (27:19), "*As water **reflects** the face, so one's life **reflects** the heart*." The apostle James (James 1:23-24) wrote, "*Anyone who listens to the word but does not do what it says is like someone who looks at his face in a mirror and, after looking at himself, goes away and* *immediately forgets what he looks like*." The importance of the Bible as a mirror became part of Christian thought. Augustine of Hippo (427) extracted the divine precepts from the Old and New Testaments into a single volume that he entitled *Speculum*, for those, who were not great readers, so that they could ***reflect*** on their obedience to God. Pope Gregory the Great (ca. 600) wrote, "*The Holy Bible is like a mirror before our mind's eye. In it we see our inner face. From the Scriptures we can learn our spiritual deformities and beauties. And there too we discover the progress we are making and how far we are from perfection*."

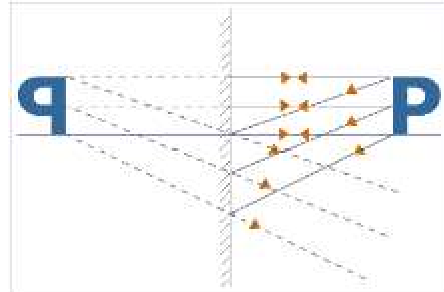
The word speculation is related to speculum. To Thomas Aquinas, speculation meant a consideration of the relationship between two subjects that could be modeled by the relationship between the object and the image produced by a mirror and described by the law of reflection.Thomas Aquinas (ca. 1250) wrote "*To see something by means of a mirror is to see a cause in its effect wherein its likeness is reflected. From this we see that 'speculation' leads back to meditation*."

The church was not always supportive of optical knowledge. In the 13th century there was a movement to protect religious orthodoxy and eliminate heresy—"*any provocative belief or theory that is strongly at variance with established beliefs or customs*." The word heresy comes from the Greek word *haireisthai* meaning to choose. **Robert Grosseteste** (1175-1253), a bishop and a scientist, whose optical research (*De Luce*) inspired Roger Bacon, defined heresy as "*an opinion chosen by human perception, created by human reason, founded on the Scriptures, contrary to the teachings of the Church, publicly avowed, and obstinately defended*." The movement to prevent questioning the orthodoxy was ironically known as the *inquistion*—which means a period of prolonged and intensive questioning (*inquisitio* is Latin for inquiry). During the inquisition, one of the questions in the *Summa de officio inquisitionis* of 1270 was aimed at finding those who practiced divination using reflective objects (catoptromancy), "*Have you conducted experiments with mirrors, swords, fingernails, spheres or ivory handles?*"

The mirror was a very powerful symbol—being capable of reflecting the truth and of producing illusions. The German folk legend of **Till Eulenspiegel** (*eulen* and *spiegel* are the German words for owl and mirror, respectively) describes a trickster born around 1300 who carried a mirror and an owl as he traveled through the countryside exposing injustice, lunacy and hypocrisy. Richard Strauss put Till Eulenspiegel's story to music. In the beginning of *Til Eulenspiegel's Merry Pranks,* the music tells of his trips through the countryside poking fun at and mocking the establishment. The end tells us of his capture by the authorities, his death sentence for blasphemy, and his execution by hanging. After his death, the playful initial musical theme reappears, suggesting that Till Eulenspiegel's message will live forever. https://www.youtube.com/watch?v=S7O9Oa22nsQ

Even today, various mirrors tell us about ourselves and the world around us. In analogy to silvered glass mirrors, we ask, do these newspapers and magazines reflect the truth or produce illusion?

We can determine the position, orientation, and size of an image formed by a **plane mirror** by drawing light rays radiating from at least two different points on the object to the mirror. Then we assume that rays that strike the mirror are reflected in such a way that the angle of reflection equals the angle of incidence. Practically, we find a given image point by drawing two of the infinite number of rays that radiate from a given point on the object. We draw these rays, which are known as **characteristic rays**, using the following rules:

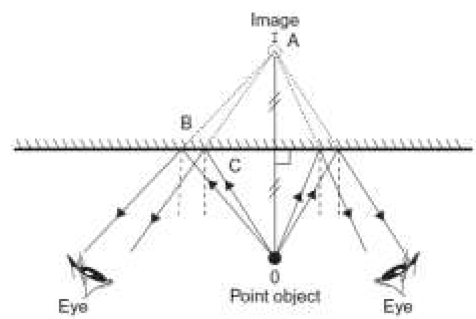


1. From a given point on the object, draw a line perpendicular to the mirror. Because $\theta_i = 0$ then $\theta_r = 0$. Draw the reflected ray and then extend the reflected ray backwards behind the mirror.

2. From the same point on the object, draw another line to any other point on the mirror. Draw the normal to the mirror at this point and then draw the reflected ray using the rule $\theta_r = \theta_i$. Extend the reflected rays backwards behind the mirror, to the other extended reflected ray originating from the same point on the object. The point of **intersection** of the extensions of the reflected rays originating from the same object point is the position of the image of that object point. If the reflected rays **converged** in front of the mirror, which they do not do when they strike a plane mirror, a **real image** would have been formed. A real image is an image that can be projected on a piece of translucent vellum. A real image is composed of radiant energy, and the light intensities of the points that make up a real image can be measured with a light

meter. However, since the reflected rays **diverge** from a plane mirror, we extend the rays backwards from where they appear to be diverging. While an image appears in the place from which the rays appear to diverge, a piece of translucent vellum would ***not*** display an image. The image is **virtual**—it appears only in our mind's eye.

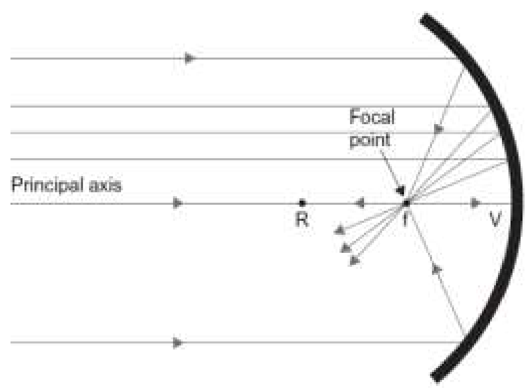
Oddly enough, we can resurrect the **extramission theory** to help us understand the placement of the virtual image. The mind's eye assumes that the diverging rays originate from a single point behind the mirror. This is exactly where the **visual rays** issuing ***from*** the eye would converge. Of course there are no visual rays and in fact light from the object actually enters our eyes after it bounces off the mirror. However, the mind knows where it sees the virtual image because of the **neural signals** that travel between our **neck**, **head** and **eye muscles** and our **brain** to let our mind know the direction the eyes are looking in order to see the virtual image.



There is a story told in many cultures about a man who, in ancient times, when mirrors were rare, brought home a mirror as a present for his wife. She looked into the mirror and saw another woman who she immediately assumed was her husband's lover and started yelling at her husband. He looked into the mirror not knowing what his wife was talking about and saw another man who he assumed was his wife's lover. A huge ruckus commenced. All I want to say is, learning the laws of geometrical optics may be good for your relationship!

Not all mirrors are planar. We will look at images produced by **concave** and **convex mirrors**.

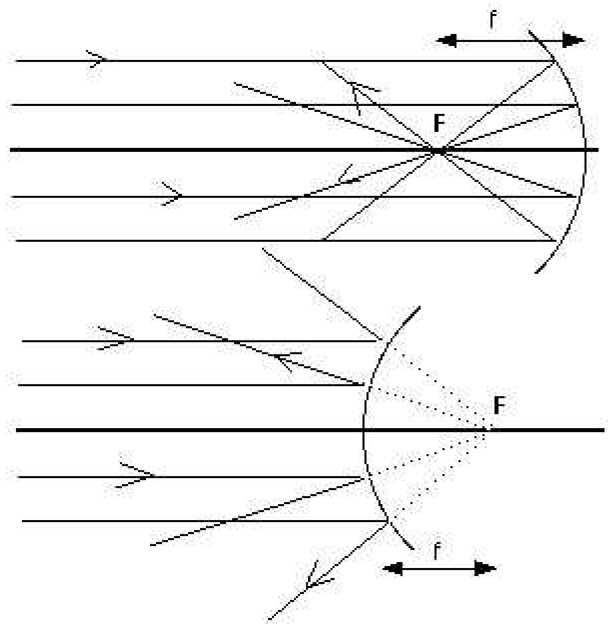
Demonstration: Place a short candle 40-50 cm from the **convex** mirror and slowly move it toward the mirror and away from you. Describe what happens to the image of the candle as its distance to the convex mirror decreases. Place the candle 40-50 cm from the **concave** mirror and slowly move it toward the mirror and away from you. Describe what happens to the image of the candle as its distance to the concave mirror decreases. Compare the images made by the concave and the convex mirrors. Place the candle approximately 30 cm from the concave mirror. Find the image with the translucent vellum. Move the candle towards the mirror. Find the image with the translucent vellum. Move the candle away from the mirror. Find the image with the translucent vellum. How does the image formed by the concave mirror on the translucent vellum change with the position of the object?



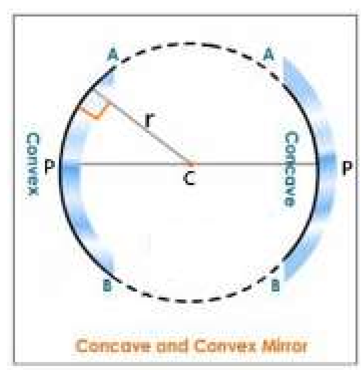
The **center of curvature** of a mirror is defined as the center of the imaginary sphere of which the curved mirror would be a part. The distance between the center of curvature (C) and the mirror itself is known as the radius of curvature (R). For a concave mirror, the radius of curvature is on the object side of the mirror and for a convex mirror, the radius of curvature is on the image side.

The line connecting the midpoint of the mirror with the center of curvature is called the **principal axis** of the mirror. The **vertex** V represents the intersection of the mirror and the principal axis. The **focus**, which is **positive** for a **concave mirror**, is on the principal axis midway between the mirror and the center of curvature. The focal length of a mirror is the distance between the focus and the vertex.

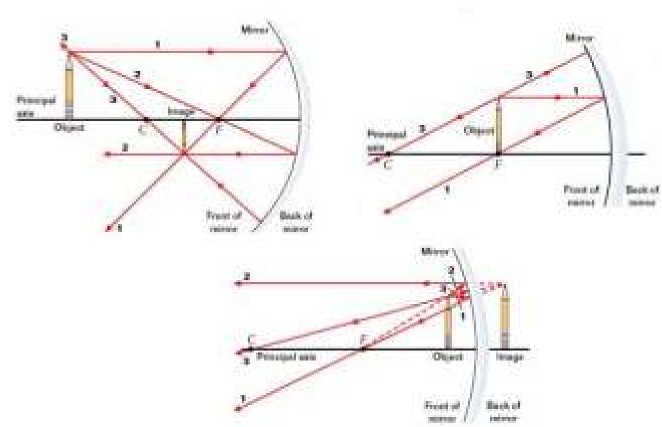
Consider a beam of light that strikes the mirror **parallel to the principal axis**. When a ray of light in this beam moves along the principal axis and strikes the mirror, it is reflected back on itself. When a ray of light in this beam strikes the mirror slightly above or below the principal axis, the ray makes a small angle with the normal and consequently the reflected ray is bent slightly toward the principal axis. If the incident ray strikes the mirror farther away from the principal axis, the reflected ray is bent toward the principal axis with a greater angle. In all cases, the reflected rays from every part of the mirror converge toward the **focus**.



Moreover, a line drawn between the center of curvature and the point on the mirror where a ray parallel to the principal axis strikes bisects the angle that subtends the incident and reflected rays. Since the line that comes from the center of curvature is radial, it is perpendicular to the surface of the mirror and in all cases, $\theta_r = \theta_i$ and the law of reflection holds.



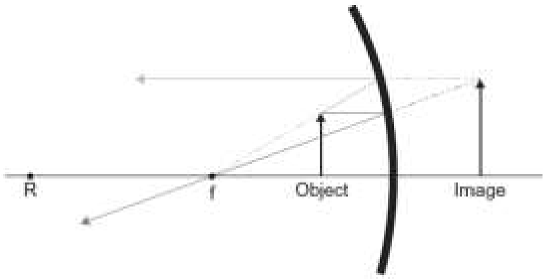
While the position, orientation, and size of the image formed by a **concave mirror** can be determined by any two of the infinite number of rays that radiate from each object point, they are most easily determined by drawing two or three **characteristic rays** that are based on the law of reflection from at least two points on the object (the point on the principal axis is a giveaway):

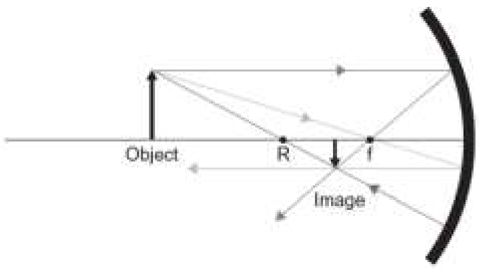


1. A ray traveling parallel to the principal axis passes through the focus (f) after striking a concave mirror.
2. A ray that travels through the focus on the way to the concave mirror or appears to come from the focus, if the object is between the focus and the mirror, travels backwards parallel to the principal axis after striking the mirror.
3. A ray that passes through the center of curvature (R) is reflected back through the center of curvature.

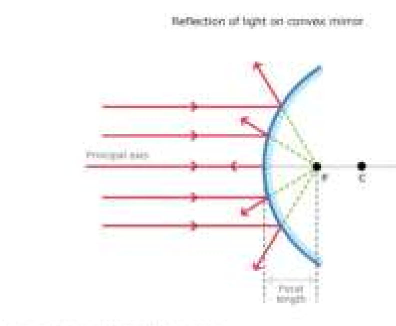
A **real image** of an **object point** is formed at the point where the rays **converge**. If the rays do not converge at a point, a **virtual image** may be formed. To find the virtual image of an object point, one must trace back the ***reflected*** rays to the point from which the extensions of each reflected ray seem to **diverge**. Tracing the reflected rays backward is geometrically equivalent to tracing the visual rays forward.

Here are examples of image formation by a concave mirror when the object distance is less than or greater than the focal length, respectively.

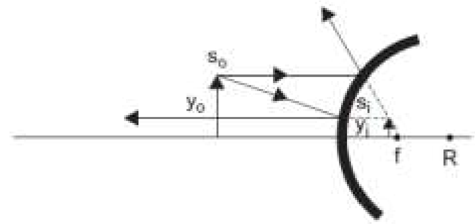


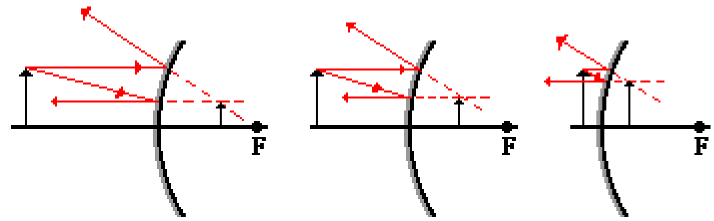


Convex mirrors, by convention, have a positive radius of curvature. When a beam of light parallel to the principal axis strikes a convex mirror, the rays are reflected away from the principal axis, and therefore **diverge**. If we follow the reflected rays backward, they appear to originate from a point behind the mirror, known as the **focus**. Since the focus is behind the mirror, ***f*** and the focal length are **negative**. We can draw a line between the center of curvature and the point on the mirror where a ray parallel to the principal axis strikes. This line, which is perpendicular to the surface of the mirror, bisects the angle that subtends the incident and reflected rays so that in all cases, $\theta_r = \theta_i$ and the law of reflection holds.



We can determine the position, orientation, and size of the image formed by a **convex mirror** by drawing two or three **characteristic rays,** from at two points on the object, that are based on the law of reflection (the point on the principal axis is a giveaway):





1. A ray traveling parallel to the principal axis is reflected from the convex mirror as if it originated from the focus (f).

2. A ray that travels toward the focus on the way to the mirror is reflected back parallel to the principal axis after striking the mirror.

3. A ray that strikes the mirror as it was heading toward the center of curvature (R) is reflected back along the same path.

A real image of an object point is ***never*** formed by a convex mirror. If we trace back the reflected rays to a point from where the extensions of each reflected ray seem to **diverge**, we will find the **virtual image** of the object point. The virtual image is erect and **minified**. The minified image makes us think that the objects are farther than they appear, but really, as it says in the side view (convex) mirror: "*objects in the mirror are closer than they appear*."

As an alternative to drawing characteristic rays known **as ray tracing**, we can determine **analytically** with the aid of algebra, where the reflected rays originating from a luminous or nonluminous object will converge (or seem to diverge) to form an image. The formula we use is known as the **Gaussian lens equation**:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

where s_o is the distance from the object to the mirror (in m), s_i is the distance between the image and the mirror (in m), and f is the focal length of the mirror (in m). The magnification (m_T) is defined as ($\frac{y_i}{y_o}$), where y_i and y_o are linear dimensions (in m) of the image and object, respectively, is given by the following formula:

$$m_T = \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

When using these formulae for concave and convex mirrors, which will be discussed below, the following sign conventions must be observed: s_o, s_i and f are positive when they are in front of the vertex (V) of the mirror and negative when behind; and y_i and y_o are positive when they are above the principal axis and negative when below.

When s_i is **positive**, the image formed by a concave mirror is **real**. When s_i is **negative**, the image formed by a concave mirror is **virtual**. The image is erect when m_T is positive and inverted when m_T is negative. The degree of magnification or minification is given by the absolute value of m_T. Let’s have a little practice in using the preceding formulae for **concave mirrors**:

Example 1: When an object is placed at infinity ($s_o = \infty$), $1/s_o$ equals zero, and thus $\frac{1}{s_i} = \frac{1}{f}$ and $s_i = f$. In other words, when an object is placed at an infinite distance from the mirror, the image is formed at the focal point and the magnification ($-\frac{s_i}{s_o}$) is equal to zero.

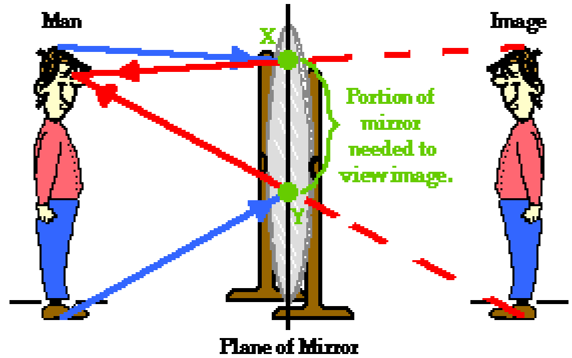
Example 2: When an object is placed at the focus (s_o= f), $\frac{1}{f} + \frac{1}{s_i} = \frac{1}{f}$. Thus $\frac{1}{s_i}$ must equal zero and s_i must be equal to infinity. In other words, when an object is placed at the focus, the image is formed at infinity, and the magnification ($-\frac{s_i}{s_o}$) is almost infinite.

Example 3: When an object is placed at the center of curvature (s_o= $2f$), then $\frac{1}{2f} + \frac{1}{s_i} = \frac{1}{f}$ and $\frac{1}{s_i} = \frac{1}{2f}$ (remember ½ - ¼ = ¼). Thus $s_i = 2f$, and the image is real and the same distance from the mirror as the object is. The magnification ($-\frac{s_i}{s_o}$) is minus one, and the image is inverted.

Example 4: When an object is placed at a distance equal to $\frac{1}{2}f$, which is between the focus and the mirror, then $\frac{2}{f} + \frac{1}{s_i} = \frac{1}{f}$. Thus $\frac{1}{s_i} = \frac{1}{f} - \frac{2}{f}$ and $\frac{1}{s_i} = -\frac{1}{f}$, and $s_i = -f$. Since s_i is a negative number, the image is behind the mirror and virtual. Since $-\frac{s_i}{s_o}$ equals +2, the image is erect and twice the height as the object.

The **Gaussian lens equation** can also be used to determine the nature of images produced by **convex mirrors**. For convex mirrors, the focal length is **negative**. To form any image in a convex mirror where f is always negative, s_o must be positive. According to the Gaussian lens equation, $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$, s_i will always be negative and the image will always be behind the mirror and virtual. Since $-\frac{s_i}{s_o}$ will always be positive, the image will always be erect.

The **Gaussian lens equation** can also be used to determine the characteristics of an image formed by a **plane mirror** analytically. When light rays parallel to the normal strike the mirror, they are reflected back along the normal and remain parallel. That is, they never converge and the focal length of a plane mirror is equal to infinity, and $\frac{1}{f} = 0$. Thus $\frac{1}{s_o} = -\frac{1}{s_i}$ and $-\frac{s_i}{s_o}$ will always be positive and equal to one. This describes the image in a plane mirror as being erect, the same size as the object, and virtual.



The following table summarizes the nature of the images formed by concave and convex mirrors:

TABLE 2.1 Nature of Images Formed by Curved Mirrors

Location	Type	Location	Orientation	Relative Size
Object		Image in a Concave Mirror		
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$			∞	
$s_o < f$	Virtual	$\lvert s_i \rvert > s_o$	Erect	Magnified
Object		Image in a Convex Mirror		
Anywhere	Virtual	$\lvert s_i \rvert < \lvert f \rvert$	Erect	Minified

For fun, which kind of mirror is depicted in each of the following paintings: *John Arnolfini and His Wife* by Jan Van Eyck, *Venus and Cupid* by Diego Rodriguez de Silva y Veláquez, *Self Portrait* by Parmigianino and *La Reproduction Interdite* by René Magritte?

John Ashbery (1974) wrote a poem *Self-portrait in a convex* (http://www.poetryfoundation.org/poetrymagazine/browse/124/5#!/20596528/0 https://www.youtube.com/watch?v=zrvXX9QVAT8). Other paintings that depict mirrors include, *Vanity* by Hans Memling; *The Moneychanger and his Wife* by Quentin Metsys; *The Bar at the Folies Bergères* by Edouard Manet; *Joking Couple* by Hans von Aachen; *Venus in Front of the Mirror* by Peter Paul Rubens; *Venus in Front of the Mirror* by Titian; *Nude Standing before a Mirror* by Henri Toulouse Lautrec; and *Triple Self Portrait* by Norman Rockwell. See http://larsdatter.com/mirrors.htm.

The set in Christopher Wheeldon's *Scènes de Ballet* is a Russian ballet studio bisected by a barre and an imaginary mirror. Sixty two dancers are divided between "real" dancers and their "reflections."

How many "real" dancers are in this freestyle dance video called *Mirror Story*? https://www.youtube.com/watch?v=HzZ1K-IRxA4

As long as we consider only the rays that emanate from a given point of an object and strike close to the midpoint of a **spherical** mirror, we will find that these rays converge at a point. However, when the incident rays hit a **spherical** mirror far from the midpoint, they will not be bent sharply enough and will not converge at the same point as the rays that strike close to the midpoint of the mirror. Thus, even though all rays obey the law of reflection, a **spherical** zone of confusion instead of a luminous point results. The **inflation of a point into a sphere** by a spherical mirror results in **spherical aberration**, from the Latin word *aberrans*, which means wandering.

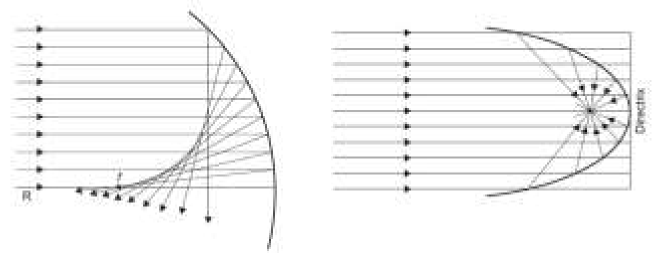


FIGURE 2.11 Because of the law of reflection, where the angle of reflection equals the angle of incidence, a spherical mirror does not focus parallel rays

While spherical mirrors are abundant since they are easy and cheap to make, Descartes found that the correct shape of a mirror that leads to a perfect focus is **parabolic**. **Chromatic aberration** occurs when parallel rays of light of different colors are not focused to the same point. While spherical aberration occurs in spherical mirrors, chromatic aberration does ***not*** occur in any mirrors.

Now we will talk about the influence of **refraction** on **image formation. Light travels in straight lines** as long as it remains in a single **homogeneous medium**, however, when a light ray traveling through air strikes a **denser medium** (e.g. water or glass) at an **oblique angle** (θ_i) with respect to the normal, the ray is bent toward the normal in the denser medium. The angle that the light ray makes in the denser medium, relative to the normal, is known as the angle of transmission (θ_t). Ptolemy found that the angle of transmission is always smaller than the angle of incidence but never discovered the true relationship.

The true mathematical relationship between the angle of incidence and the angle of transmission was first worked out by Willebrord Snel van Royen in 1621. Snel, also known as Snellius did not publish his work. René Descartes, who independently worked out the relationship, published the relationship in 1637. The law of refraction, which is known as the **Snel-Descartes Law**, states that when light passes from air to a denser medium, the ratio of the sine of the angle of incidence to the sine of the angle of transmission equal to a constant, called the **refractive index**. The Snel-Descartes Law can be expressed by the following equation:

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_t}{n_i} = n_t \text{ (when } n_i = 1)$$

where n_t is the refractive index of the denser medium and n_i is the refractive index of air (n_i = 1). The Snel-Descartes Law can be written more generally as:

$$n_i \sin\theta_i = \ n_t \sin\theta_t$$

Demonstration: Make a table of the relationship between the angle of incidence and the angle of transmission through water. Check out these two formulaic models:

$$\frac{\theta_i}{\theta_t} = n_t$$

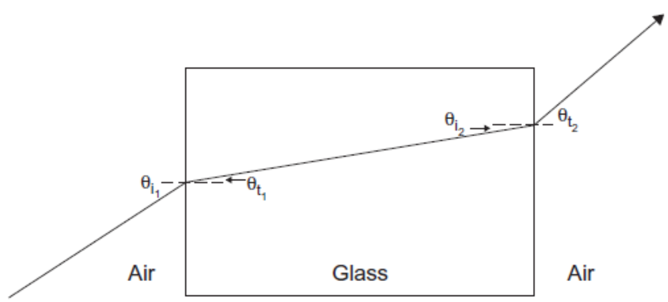
$$\frac{\sin\theta_i}{\sin\theta_t} = n_t$$

Which formula seems to be correct and why does it seem to you to be correct?

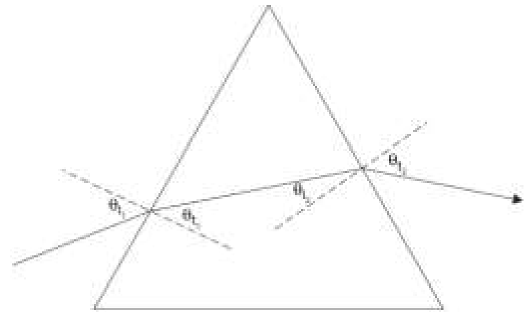
Demonstration: In general, the denser a transparent medium, the greater is its refractive index. We can observe this by measuring the refractive indices of 0%, 5% and 10% (w/v) sucrose solutions with a hand-held refractometer. The solution in the hand-held refractometer bends light in a manner that depends on the refractive index of the solution. The hand-held refractometer is useful to beer and wine makers and is often calibrated in degrees Brix instead of refractive index. Degrees Brix is equivalent to the percent sugar (*S*). The refractive index of the solution = 1.3333 + (0.0018)(*S*).

Demonstration: A piece of Pyrex glass becomes invisible when it is placed in a solution of Wesson (soybean) oil, but not in air or water. Can you guess why?

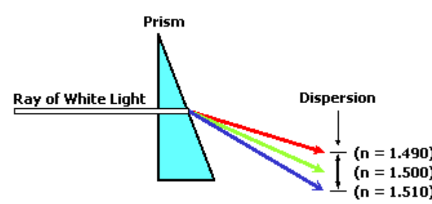
When a light ray passes from **air** through a piece of glass with **parallel edges** and returns to the **air**, the refraction at the far edge **reverses** the refraction at the near edge and the ray **emerges parallel** to the incident ray, although slightly **displaced.** The amount of displacement depends on two things: the **refractive index of the glass** and the **distance the beam travels in the glass**.



However, when the edges are **not parallel**, the refraction at the far edge will ***not* reverse** the effect of the refraction at the near edge. In this case, the light ray will not emerge parallel to the incident light ray, but will be **bent** in a manner that depends on the shape of the edges. Consider a ray of light passing through a **prism** oriented with its apex upward. If the ray of light hits the normal at an angle from below, it crosses into the glass above the normal but makes a smaller angle with respect to the normal since the glass has a higher refractive index than the air. When the ray of light reaches the glass-air interface at the far side of the prism, it makes an angle with a new normal. As it emerges into the air it bends away from the normal since the refractive index of air is less than the refractive index of glass. The result is the ray of **light is bent twice in the same direction**.

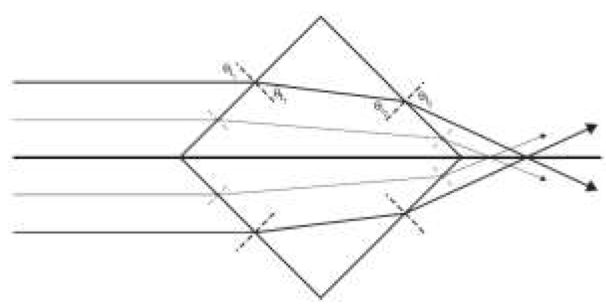


Aside: The refractive index of glass, or the amount it bends light, is not a constant but varies with the color of light. For this reason, as Isaac Newton discovered, a prism splits or **disperses** white light into its constituent colors in the form of a **spectrum**.



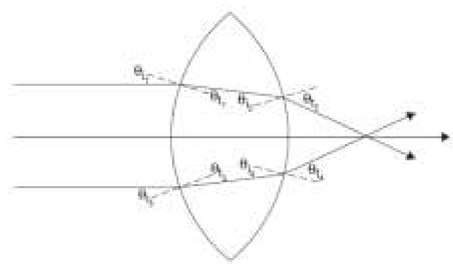
Chromatic Dispersion of White Light

What would happen to a parallel beam of incident light rays that strike two prisms whose bases are cemented together? The light that strikes the upper prism will be **bent downward** toward its base and the light that strikes the lower prism will be **bent upward** toward its base. The



two halves of the beam of light will **converge** and cross on the other side. However, the beam emerging from this double prism will not come to a focus since the rays that strike the two corresponding prisms farther and **farther from the principal axis** all strike at the same angles but travel through **less and less glass** and thus will converge at greater and greater distances from the double prism. Is there a particular shape of glass that will bring parallel rays to converge at a **focus**?

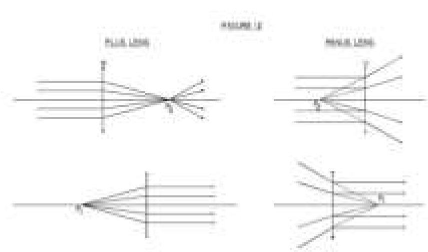
A "**lentil-shaped**" transparent object will cause parallel rays to converge at a focus according to the Snel-Descartes Law and consequently, a **lens** got its name from the Latin word for lentil. Everything is botany!!!!



The ability of a lens to bend or refract light rays is characterized by its **focal length**; the shorter the focal length, the greater the ability of the lens to bend light. A lens with a long focal length is relatively flat and a lens with a short focal length is more curved. The power of a lens to bend light is known as the **dioptric power** (D). It is given by the reciprocal of the focal length in meters:

$$D = \frac{1}{f}$$

Lenses have two **focal points**—an **object focal point** and an **image focal point**. The two focal points, which lie on the principal axis, can be found in this way: The image focal point of a **converging lens** is the place where parallel rays passing through the lens converge. The object focal point of a **converging lens** is where you place



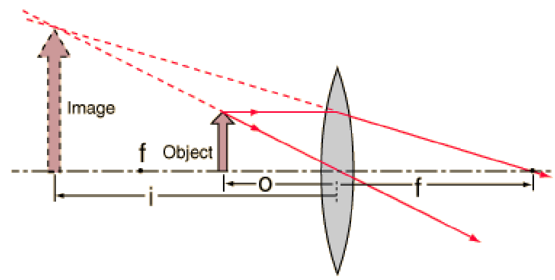
a point source of light so that the light that passes through the lens comes out as parallel rays. **Converging lenses** have **positive focal lengths**.

The image focal point of a **diverging lens** is the place where rays diverging from the lens appear to have diverged from. The object focal point is where a virtual point source of light seems to be when parallel rays of light emerge from the lens. **Diverging lenses** have **negative focal lengths.**

Demonstration: Place the bayberry candle from Monticello at one end of the desk. Look at it close-up through the 0.5 m (2 D) and then the 1 m (1 D) focal length double convex converging lenses. Describe the images formed by the two lenses. Are the images real or virtual? Erect or inverted? Which one forms the larger image? Then place the 0.5 m and 1 m focal length converging lenses side by side about 2 m away from the candle. Find the images behind the lenses by moving the translucent vellum backwards starting at the back of the lenses. Describe and compare the images that are formed by the two lenses. Are the images real or virtual? Erect or inverted? Which one forms the larger image? Place a -1 m (-1 D) focal length double concave diverging lens immediately behind the 0.5 m (2 D) double convex converging lens. Find the image with the translucent vellum. What happens to the image? Compare it to the images formed by the 0.5 m and the 1m focal length converging lenses.

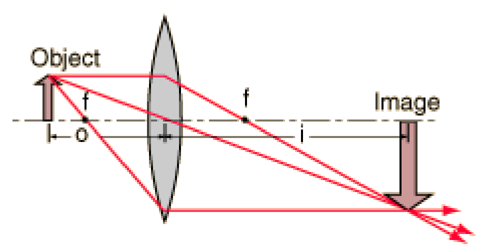
Bayberry (*Myrica pensylvanica*) grows on the patio of the Plant Sciences Bldg. Why might the berries be coated with wax? Why are plants in general, whether they live in the desert or the tropics, coated with wax?

We can characterize the type, location, orientation, and relative size of images formed by converging and diverging lenses using the ray tracing method just as we characterized the images formed by mirrors. We must draw two or three **characteristic rays** from at **least two points** that obey the Snel-Descartes Law Law (a point on the principal axis is a giveaway):



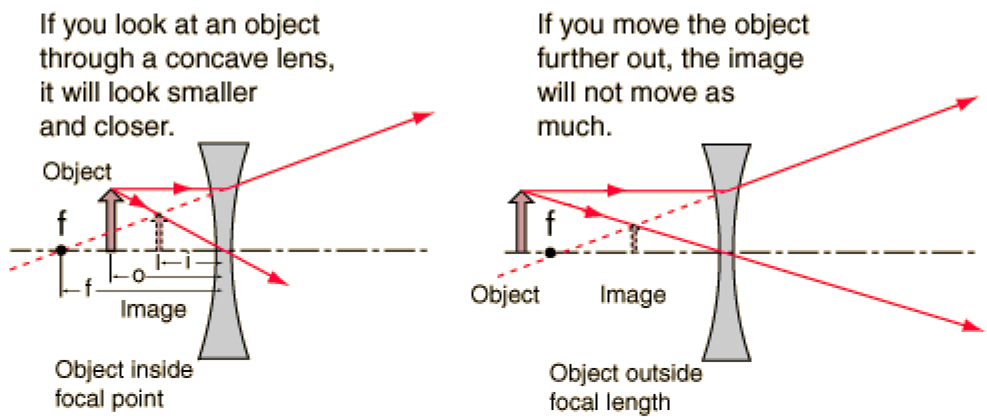
1. A ray that strikes a converging lens parallel to the principal axis goes through the image focus (f_i).
2. A ray that strikes a diverging lens parallel to the principal axis appears to have come from the image focus (f_i).
3. A ray that strikes a converging lens after it passes through the object focus (f_o) emerges parallel to the principal axis.
4. A ray that strikes a diverging lens on its way to the object focus (f_o) emerges parallel to the principal axis.
5. A ray that passes through the vertex (V) of a converging or diverging lens passes through undeviated.

Draw three characteristic rays from each point on the object when the object is **farther than the object focal length** of a **converging lens**. The rays converge and the image is real, magnified and inverted. As the object moves farther away from the focus, the image goes from magnified to same size to minified.



Draw three characteristic rays from each point on the object when the object is **closer than the object focal length** of a converging lens. Since the rays do not converge, we have to trace them back and see where they appear to diverge from. The image is erect, virtual and magnified. As the object moves toward the focus, the magnification increases.

Draw three characteristic rays from each point on the object when the object is anywhere relative to the image focal point of a **diverging lens**. Since the rays do not converge, we have to trace them back and see where they appear to diverge from. The image is virtual, erect and minified. As the object is moved farther from the lens, the image will be more and more minified.



As an **alternative** to **drawing** characteristic rays, we can determine **analytically** with the aid of algebra, where the refracted rays originating from a luminous or nonluminous object will converge to form an image. We use the same **Gaussian lens equation** we used for finding the images formed by mirrors:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

where s_o is the distance from the object to the lens (in m), s_i is the distance between the image and the lens (in m), and f is the focal length of the lens (in m). A **converging lens** has a **positive focal length** (f) and a **diverging lens** has a **negative focal length** (f). s_o is positive for a real object in front of the lens. s_i is positive for a real image behind the lens and is negative for a virtual image in front of the lens. The magnification (m_T) is defined by ($\frac{y_i}{y_o}$), where y_i and y_o are linear dimensions (in m) of the image and object, respectively. The magnification is given by the following formula:

$$m_T \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

y_i and y_o are positive when they are above the principal axis and negative when they are below the principal axis. When $m_T > 0$, the image is virtual and erect and when $m_T < 0$, the image is real and inverted.

Example 1: When an object is placed at infinity (s_o= ∞) in front of a converging lens, $1/s_o$ equals zero, and thus $\frac{1}{s_i} = \frac{1}{f}$ and $s_i = f$. In other words, when an object is placed at an infinite distance from a converging lens, the image is formed at the focal point and the magnification ($-\frac{s_i}{s_o}$) is equal to zero.

Example 2: When an object is placed at the focus (s_o= f) of a converging lens, $\frac{1}{f} + \frac{1}{s_i} = \frac{1}{f}$. Thus $\frac{1}{s_i}$ must equal zero and s_i must be equal to infinity. In other words, when an object is placed at the focus of a converging lens, the image is formed at infinity, and the magnification ($-\frac{s_i}{s_o}$) is almost infinite.

Example 3: When an object is placed at the center of curvature of a converging lens (s_o= $2f$), then $\frac{1}{2f} + \frac{1}{s_i} = \frac{1}{f}$ and $\frac{1}{s_i} = \frac{1}{2f}$ (remember ¼ + ¼ = ½). Thus $s_i = 2f$, and the image is real and the same distance from the lens as the object is. The magnification ($-\frac{s_i}{s_o}$) is minus one, and the image is inverted.

Example 4: When an object is placed at a distance equal to $\frac{1}{2}f$ in front of a converging lens, which is between the focus and the lens, then $\frac{2}{f} + \frac{1}{s_i} = \frac{1}{f}$. Thus $\frac{1}{s_i} = \frac{1}{f} - \frac{2}{f}$ and $\frac{1}{s_i} = -\frac{1}{f}$, and $s_i = -f$. Since $s_i$is a negative number, the image is is on the same side of the lens as the object, and it is virtual. Since $-\frac{s_i}{s_o}$ equals +2, the image is erect and twice the height as the object.

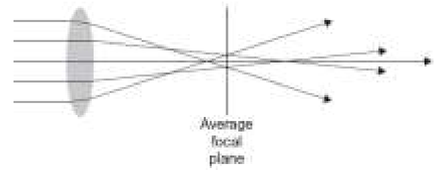
Example 5: When an object is placed at a positive distance equal to $-\frac{1}{2}f$ (remember that f is negative) in front of a diverging lens, which is between the focus and the lens, then $-\frac{2}{f} + \frac{1}{s_i} = \frac{1}{f}$. Thus $\frac{1}{s_i} = \frac{1}{f} - \frac{-2}{f}$ and $\frac{1}{s_i} = \frac{3}{f}$, and $s_i = \frac{f}{3}$. Since s_i is a negative number, the image is is on the same side of the lens as the object, and it is virtual. Since $-\frac{s_i}{s_o}$ equals $+\frac{2}{3}$, the image is erect and two-thirds the height as the object.

The following table summarizes the nature of the images formed by converging and diverging lenses.

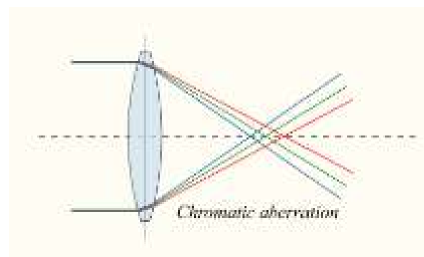
TABLE 2.5 Nature of Images Formed by Lenses

Location	Type	Location	Orientation	Relative Size
Object		**Image Formed by a Converging Lens**		
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		∞		
$s_o < f$	Virtual	$\|s_i\| > s_o$	Erect	Magnified
Object		**Image Formed by a Diverging Lens**		
Anywhere	Virtual	$\|s_i\| < \|f\|$	Erect	Minified

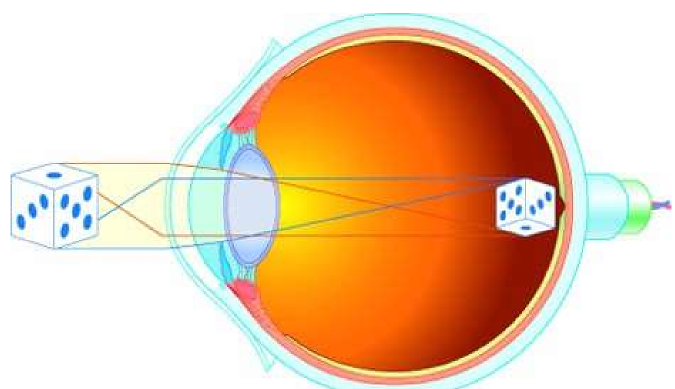
Refracting lenses can have both spherical and chromatic aberrations. **Spherical aberration** occurs because the rays from any given object point that hit a lens with spherical surfaces far from the principal axis are refracted too strongly, resulting in the inflation of a point into a sphere. Spherical aberration can be reduced by molding or grinding the lens so that it has aspherical surfaces or using only the part of the lens close to the principal axis.



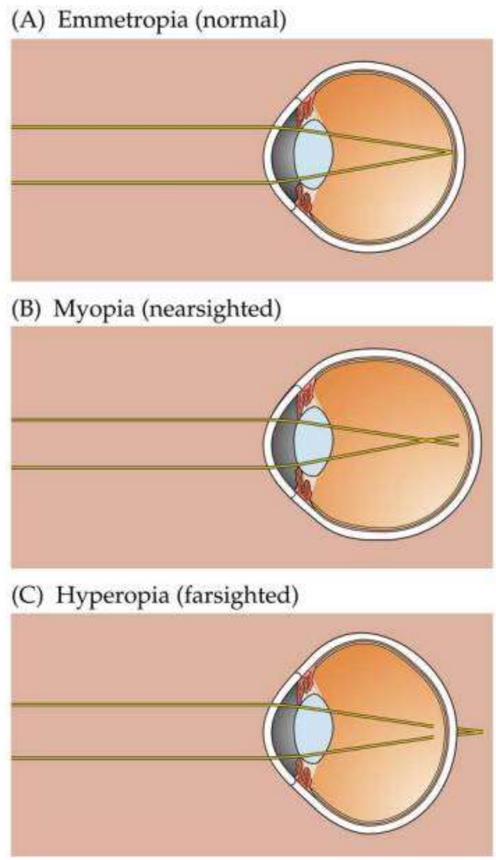
Chromatic aberration occurs because the refractive index of refracting materials is color-dependent. This results in the violet-blue rays being more strongly refracted by glass than the orange-red rays and the image has colored halos.



The **eye** too has some spherical and chromatic aberration but it is not noticeable to us. Perhaps a small pupil minimizes these aberrations. Children have larger pupils than adults, indicating that the aberrations may get worse with age. Together, the cornea and the crystalline lens act as a converging lens that produces a real reliable minified image on the retina (**emmetropia**).

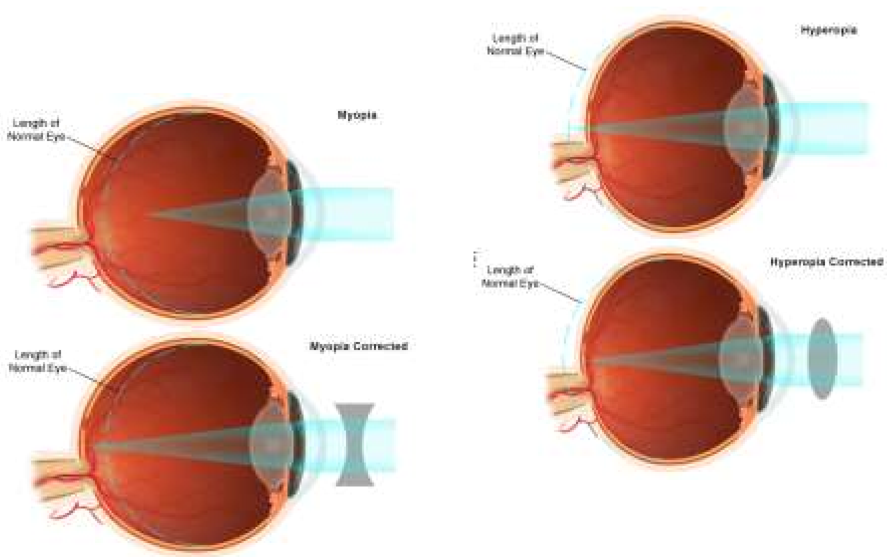


If the focal length of an eye is **too short**, and the dioptric power is too high because the cornea is **too convex** (or the eyeball **too long**), the image will be formed in front of the retina and vision will be nearsighted (**myopia**). Myopia can be corrected by using spectacles with diverging lenses with negative focal lengths and negative dioptric powers. Myopia can also be corrected with Lasik surgery that makes the cornea less convex.

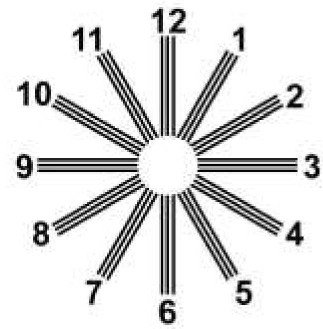


If the focal length of an eye is **too long**, and the dioptric power is too low because the cornea is not convex enough (or the eyeball **too short**), the image will be formed behind the retina and vision will be farsighted (**hyperopia**). Hyperopia can be corrected by using spectacles with converging lenses with positive focal lengths and positive dioptric powers. Hyperopia can also be corrected with Lasik surgery that makes the cornea rounder.

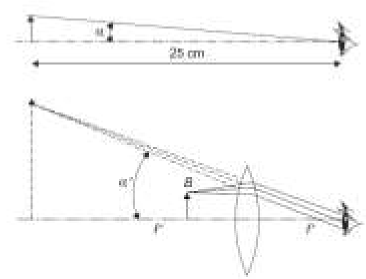
The effect of spectacles on myopia (nearsightedness) and hyperopia (farsightedness).



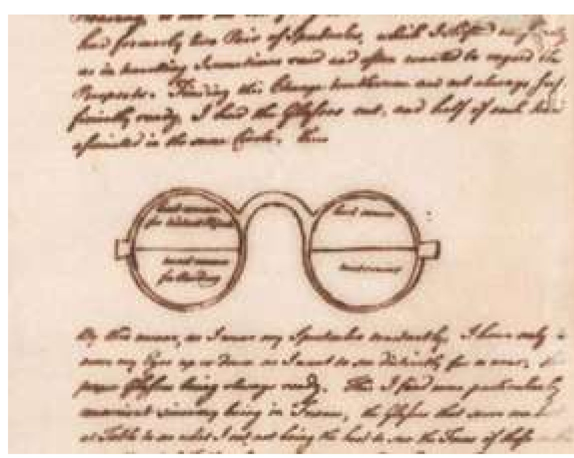
The cornea of some eyes are not symmetrical and the focal length in one radial direction is greater than the focal length in another radial direction. For example in the **astigmatism** test on the right, the lines marked with a 3 may be in focus but the lines marked with a 12 are not. Astigmatism can be corrected with **cylindrical lenses** or with Lasik surgery.



The **cornea** is fixed and can only focus distant objects on the **retina**. The **crystalline lens** is elastic and can change its shape to focus near and far objects on the retina. The ciliary **muscles** contract (**accommodate**) to focus near objects and relax to focus distant objects. As one ages, the crystalline lens loses it elasticity and the ability to accommodate decreases. This is called **presbyopia** and it is corrected with reading glasses made with converging lenses. As Roger Bacon realized, a converging lens used as a magnifying glass increases the angle of the light rays that reach the eye for old men!!!



Bifocals are spectacles that contain lenses with two **focal lengths**. Bifocals were invented by **Benjamin Franklin** and are useful for people with **presbyopia** and either near- or far-sightedness.



Speaking of two prescriptions, *Anableps* is the four-eyed fish that lives on the surface of the water. It needs one kind of lens to look for food in the air (n=1) and another kind of lens to watch for predators in the water (n = 1.333) below.

Anyone who wears glasses knows that the **disagreeable distortions and aberrations** that you first see disappear, not from any change in the glasses themselves, but from a change in the perceiving mind. **The mind knows how to align the perceived image so that it conforms to reality**. Our brain is quite good when it comes to vision, but it still can't fix this:

Next lecture we will study the lenses of the eye—but don't forget that the shiny surface of the eye can act as a mirror and reflect the scenery. In fact, as megapixel cameras become more prevalent, zooming in on reflected images in a person's eye may help solve crimes.